A Local Realistic Reconciliation of the EPR Paradox

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Introduction

The exact CHSH violation of Bell's Inequalities by 2√2 is obtained with a local realistic model for spin. The EPR data is simulated one coincidence at a time as a product state. Such a spin is represented by operators

$$(I, \sigma_x, i\sigma_y, \sigma_z) \qquad (1)$$

in its body frame rather than the usual set of $(I, \sigma_X, \sigma_Y, \sigma_Z)$ in the laboratory frame. This endows spin with two dimensional structure with two axes of spin quantization, one in the *x* direction and the other in the *z* direction. In the absence of a measuring probe new pure states are found and these hidden states account for the quantum correlation between EPR pairs.

However, due to the non-commutation of σ_x and σ_z, the Heisenberg Uncertainty principle states that only one of the two spin axes can be measured in one experiment. In the process, half the correlation is missed.

Since there is no experimental way to confirm that two axes exist, rather than one, the choice between local realism and non-local indeterminism is subjective. Since non-locality is the basis of "quantum weirdness", Occam's razor takes the side of locality.

The usual point particle description of spin is recovered in the presence of a probe.

The EPR Paradox

In 1935 Einstein, Podolsky and Rosen (EPR) published a paper in which they asserted that quantum mechanics is incomplete.

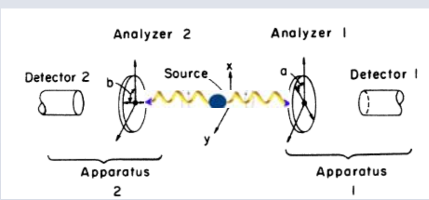
Using an entangled state, they showed position and momentum could be simultaneously deduced making them both elements of physical reality.

Since quantum mechanics describes position (momentum) but not momentum (position) EPR deduced quantum mechanics is incomplete.

In this poster it is shown that quantum mechanics can be extended to the realm below measurement to hidden variable states that complete quantum mechanics.

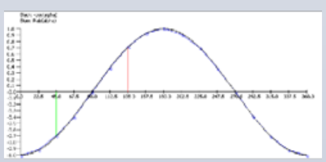
Two spins of ½ form an EPR pair.

The EPR experiment



Entangled spins in a singlet state separate into EPR pairs and are filtered with angles a at Alice and *b* at Bob. Simultaneous clicks are called coincidences and are related to the correlation by the number of coincidences that are **detected** in possible states:

$$E(a,b) = \frac{N^{++} + N^{--} - N^{+-} - N^{-+}}{N^{++} + N^{--} + N^{+-} + N^{-+}} \to -\cos\theta_{ab} \qquad (2)$$

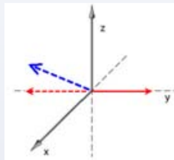


The data agrees with quantum mechanics giving the correlation as $-\cos\theta_{ab}$

The simulation shows that each axis of quantization carries exactly half the correlation.

Two dimensional spin

Equation (1) can be depicted as:



Notes:

- This is one spin that makes up a quantum ensemble.
- Each spin can be oriented differently and has its own Body Fixed Frame.
- The *x* and the *z* axes are orthogonal magnetic components. If space is isotropic (no measuring probe), the two axes are indistinguishable forming a new resonant state, in blue, that bisects the quadrant.
- The component $i\sigma_y$ is a phase that orients the *x* and *z* components is 3D space.

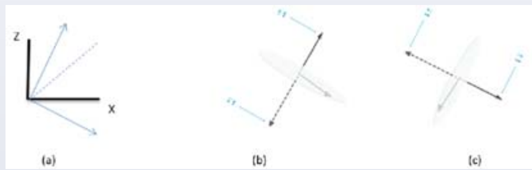
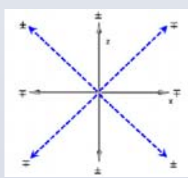


FIGURE A: Depiction of loss of correlation in the presence of a probe:
(a) Two spin axes z and x (heavy lines). The arrows depict two orientations of a probe in the laboratory frame, one closer to z and other closer to x (only one can exist at any instant).
(b) The case when the probe is closer to the z axis. The 2D spin deterministically nutates until the z axis aligns with the probe while the x component precesses in the XY plane.
(c) The same as case (b) but now the probe is closer to the x axis which aligns in the X direction while the z component precesses in ZY plane.

Notes:

- In the presence of a probe, one axis feels a greater torque than the other, and so one axis will align with the probe while the other precesses in the orthogonal plane.
- This randomizes half the correlation and results in the usual point particle view of intrinsic angular momentum.

There are four quadrants and in isotropy a spin can have equivalent states in each:



Notes for FIGURE B:

- This is not superposition but resonance. Each spin can be in one only one quadrant at any instant.
- These states give an effective magnetic monopole in the absence of a probe.
- The spin magnitude is changed from $\frac{1}{2}$ to $\frac{1}{\sqrt{2}}$.
- These states can never be measured.

Local Hidden Variables:

- The quadrants: $(n_z, n_x) = (\pm 1, \pm 1)$
- Spin orientation relative to laboratory frame: θ, ϕ

Equations:

The spin state for Alice with diagonal eigenvalues and off-diagonal quantum coherences in the z Rep:

$$s^A(n_{n_z,n_x}) = \frac{1}{2}\left(I + \mathbf{n}_{n_z,n_x}\cdot\boldsymbol{\sigma}^A\right) = \frac{1}{2}\left[\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} + \frac{1}{\sqrt{2}}\begin{pmatrix}n_z & n_x\\ n_x & -n_z\end{pmatrix}\right]$$

Unit vector that bisects each quadrant:

$$\mathbf{n}_{n_z,n_x} = \frac{1}{\sqrt{2}}\left(n_z\hat{\mathbf{z}} + n_x\hat{\mathbf{x}}\right)$$

Eigenstates and eigenvalues from diagonalizing the second matrix giving a superposition of the z and the x states:

$$(n_z\sigma_z + n_x\sigma_x)\left|\pm\sqrt{2}\right\rangle_{n_z,n_x} = \pm\sqrt{2}\left|\pm\sqrt{2}\right\rangle_{n_z,n_x}$$

$$\left|\pm\sqrt{2}\right\rangle_{+1,+1} = \frac{1}{\sqrt{(2\mp\sqrt{2})}}\left(\left|-,x\right\rangle \pm \left|-,z\right\rangle\right) \qquad \left|\pm\sqrt{2}\right\rangle_{-1,-1} = \frac{1}{\sqrt{(2\pm\sqrt{2})}}\left(\left|-,x\right\rangle \mp \left|-,z\right\rangle\right)$$

$$\left|\pm\sqrt{2}\right\rangle_{+1,-1} = \frac{1}{\sqrt{(2\mp\sqrt{2})}}\left(\left|+,x\right\rangle \mp \left|-,z\right\rangle\right) \qquad \left|\pm\sqrt{2}\right\rangle_{-1,+1} = \frac{1}{\sqrt{(2\pm\sqrt{2})}}\left(\left|+,x\right\rangle \pm \left|-,z\right\rangle\right)$$

Two representations:

- Consider an operator which depends on σ_z and σ_x

$$\mathbf{A}_a^z = \sum_{m,m'=-1}^{+1} \left|m,z\right\rangle A^z_{m,m'} \left\langle m',z\right| = \frac{1}{2}\begin{pmatrix}1+a_z & a_x+ia_y\\ a_x-ia_y & 1-a_z\end{pmatrix}$$

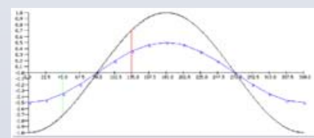
$$\mathbf{A}_a^x = \sum_{m,m'=-1}^{+1} \left|m,x\right\rangle A^x_{m,m'} \left\langle m',x\right| = \frac{1}{2}\begin{pmatrix}1+a_x & a_z-ia_y\\ a_z+ia_y & 1-a_x\end{pmatrix}$$

- The operator can be represented in either the z rep. or x rep. and their coefficients are interchanged.
- Can only measure one axis (diagonal x or z) in one experiment.
- Cannot detect the off-diagonal elements which are quantum coherence terms.

The simulation:

- A ± click at filter setting *a* is: $\left|\pm,\mathbf{a}\right\rangle$
- The states of the 2D spin are: $\left|\pm\sqrt{2}\right\rangle$
- Transform these both to the laboratory frame and calculate the matrix elements for all LHV.
- Ignore quantum coherences and take product of Alice and Bob's clicks to generate coincidences.
- Simulate one half the correlation from the *z* Rep.
- Transform to the *x* rep, repeat the calculation.
- Both give correlation shown below which fits to:

$$E(\mathbf{a},\mathbf{b}) = -\frac{1}{2}\cos\theta_{\mathbf{ab}}$$



- The sum of the two give the quantum result.

Compare with usual spin ½, $\mathbf{P}$ is the Bloch vector:

$$\rho = \frac{1}{2}(I + \mathbf{P}\cdot\boldsymbol{\sigma}) = \frac{1}{2}\left[\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} + \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\right]$$

Eigenstates and eigenvalues from the second matrix are the usual spin states:

$$\sigma_z\left|\pm1,z\right\rangle = \pm\left|\pm,z\right\rangle$$

$$\left|+,z\right\rangle = \begin{pmatrix}1\\0\end{pmatrix}, \left|-,z\right\rangle = \begin{pmatrix}0\\1\end{pmatrix}, \left|\pm,x\right\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\ \pm1\end{pmatrix}$$

The spin state for Bob, (same LHV as Alice but opposite sign to conserve angular momentum):

$$s^B(n_{n_z,n_x}) = \frac{1}{2}\left(I - n_{n_z,n_x}\cdot\sigma^B\right)$$

Bell's theorem

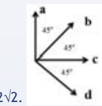
In 1964 Bell derived his inequalities that that all classical systems obey.

The CHSH form of Bell's Inequalities is:

$$\left|E(\mathbf{a},\mathbf{b}) - E(\mathbf{a},\mathbf{d}) + E(\mathbf{b},\mathbf{c}) + E(\mathbf{c},\mathbf{d})\right| \le 2$$

Correlation between spins from quantum mechanics is:

$$E(\mathbf{a},\mathbf{b}) = -\cos\theta_{\mathbf{ab}}$$



Compare with FIGURE B.

Using the filer angles as shown in the figure, quantum mechanics appears to violate the CHSH form giving 2√2.

The extra correlation is believed to come from long range entanglement, also called non-locality. Bell's theorem follows that any LHV theory that completes quantum mechanics must be non-local.

This work shows Bell's Theorem is incorrect and restores local reality to Nature.

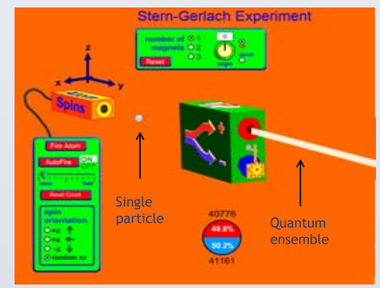
One spin in a quantum ensemble

Spin has no classical analogue and is believed to be a point particle.

Spin was discovered in the Stern-Gerlach experiment when a beam of silver atoms passes through an inhomogeneous magnetic field.

Great simulation at:

http://phet.colorado.edu/en/simulations/stern-gerlach



Quantum mechanics cannot predict if a single particle will be deflected up or down.

Only after a statistically large number of spins has passed the Stern-Gerlach filter does quantum mechanics apply.

This makes quantum mechanics a statistical theory of measurement.

In the treatment here, the individual particles that make up the quantum ensemble are treated before measurement.

Reconciliation of 2√2

- The two axes do not commute and cannot be measured simultaneously.
- From FIGURE A half the correlation is randonized on measurement.
- From Equations the diagonal elements are eigenvalues and the off-diagonal elements are quantum coherence.
- A unitary transformation can switch the diagonal and off-diagonal terms.
- Calculation in the z Rep. gives half the correlation and a second calculation in the x Rep. gives the other half.
- Each axis contributes a maximum of √2 correlation therefore each experiment satisfies Bell's Inequalities. The sum give 2√2.
- Equation (2) refers to the case of one axis.
- When there are two axes, each EPR pair is capable of two simultaneous coincidences: one from the x axis and one from the z. Therefore only half the coincidences can be detected.
- The experimental correlation must be changed from Equation (2) to:

$$E(a,b) = \frac{N^{++} + N^{--} - N^{+-} - N^{-+}}{2\left(N^{++} + N^{--} + N^{+-} + N^{-+}\right)} \to -\frac{1}{2}\cos\theta_{ab}$$